

## REFERENCES

- [1] ABBOTT, EDWIN A (1884) **Flatland: A Romance in Many Dimensions.** Available free online: <a href="http://books.google.com/books/about/Flatland.html?id=R6E0AAAAMAAJ">http://books.google.com/books/about/Flatland.html?id=R6E0AAAAMAAJ</a>
  This civil war era satirical novella not only humorously (and creepily) pokes fun at some social and political logic of the time (especially regarding gender roles), but it also provides a good discourse on the difficulty that engineers encounter dealing with the mathematics of geometry in higher dimension than we live in as human beings.
- [2] ABRAHAM, R., J.E. MARSDEN, and T. Ratiu (1988) Manifolds, Tensor Analysis, and Applications, 2nd Ed. (Applied Mathematical Sciences, Vol. 75). Springer, N.Y.
- [3] ARFKEN, GEORGE B. and HANS, J. WEBER (1995) **Mathematical Methods for Physicists**, Academic Press, San Diego.

  This is a standard reference for physicists. It provides brief and lucid overviews of many useful theorems in applied mathematics. If you find this reference too advanced, try first reading Wylie & Barrett, but be sure to come back and compare what they say with what this book says to solidify your understanding.
- [4] ARIS, R. (1962) Vectors, Tensors, and the Basic Equations of Fluid Mechanics. Englewood Cliffs, N.J.: Prentice-Hall.
- [5] BACKUS, G. (1970) A geometrical picture of anisotropic elastic tensors, *Rev. Geophys. Space GE*, **Vol. 8**, pp. 633-671.
- [6] BEYER, WILLIAM H. (2000) The CRC handbook of mathematical sciences, 6th Edition., CRC Press, Boca Raton, FL.
- [7] BIGONI, D. (2012) Nonlinear Solid Mechanics, Cambridge University Press, New York, NY.
- [8] BIRD, R.B., R.C. ARMSTRONG, and O. HASSAGER (1987) **Dynamics of Polymeric Liquids, Fluid Mechanics**, 2nd Ed, Wiley-Interscience.
- [9] BOCCARA, NINO (1990) Functional Analysis: An introduction for Physicists, Academic Press.
- [10] BOWEN, R.M. and WANG, C.C. (1976) Introduction to Vectors and Tensors, Vol. 1, Plenum, N.Y.
- [11] BJORCK, A. and C. BOWIE (1971) An iterative algorithm for computing the best estimate of an orthogonal matrix, *SIAM J. Numer. Anal.*, **8**, pp. 358-364.

  This paper provides the foundation for the iterative polar decomposition on page 604 here in our book. The B&B algorithm has stretch convergence limits, but our scaling in STEP 3 (which was *not* mentioned by B&B, but is proved in Ref. [15]) makes the algorithm converge for *all* invertible [F] matrices, no matter how large the stretch. When the cost of computing a convergence test is taken into consideration, we find B&B's method to be faster and equally accurate as Higham's [50] Newton solver (we have not tested Dubrille's [34] recent refinements of the convergence test for Higham's method, which might make Higham's iterator equally fast).
- [12] Brannon, R.M., J.A. Burghardt, S.J. Bauer, D.R. Bronowski (2009) Experimental assessment of unvalidated assumptions in classical plasticity theory, *Sandia National Laboratories Report SAND2009-0351*.
  - Available at http://www.osti.gov/bridge/
  - This government report uses discretely measured stress-strain data to infer tangent stiffness tensors, where it is pointed out that such inferences entail irreducible uncertainty since the act of measuring the response to one stimulus loading increment generally involves irreversible changes in the microstructure, precluding knowing with certainty what the response would have been to a linearly independent loading increment. This observation provides a strong argument favoring investment in mesoscale modeling (where it actually is possible to reset the material state).
- [13] Brannon, R.M., and S. Leelavanichkul (2010) A multi-stage return algorithm for solving the classical damage component of constitutive models for rocks, ceramics, and other rock-like media. *Int. J. Fract.* 163, pp. 133-149.
  - This paper illustrates pathological attributes in commonly used yield functions, and it provides three simple verification problems for verification testing classical nonhardening plasticity models.
- [14] Brannon, R. M., http://www.mech.utah.edu/~brannon/public/curvilinear.pdf
  This is an appropriate choice for follow-up reading to learn how tensor algebra formulas change in structure (not meaning) when a non-orthogonal, non-normalized, and/or non-right-handed basis system is used. This book explains the generalizations that must be included in calculus formulas whenever the basis vectors change with position (as in cylindrical or spherical coordinates).



- [15] BRANNON, R. M., "Rotation and Reflection: a comprehensive review of the theory of real orthogonal tensors in the context of engineering mechanics." A draft version is available at <a href="http://www.mech.utah.edu/~brannon/public/Rotation.pdf">http://www.mech.utah.edu/~brannon/public/Rotation.pdf</a>
  This report, which is as large as a book, explains myriad ways to describe rotations (spinors, quaternions, Euler Angles, roll/pitch/yaw, etc.), along with discussions of physical problems that require rotations (PMFI, rigid mechanics, etc.). Rotations in higher-dimensional spaces (tensor space) are covered, which has applications in plasticity theory.
- [16] Brannon, R. M. (2007) Elements of Phenomenological Plasticity: geometrical insight, computational algorithms, and applications in shock physics. Y. Horie Ed. Shock Wave Science and Technology Reference Library: Solids I, Springer-New York. 2: pp. 189-274.

  This book chapter provides a summary of classical rate-independent and rate-dependent (overstress) plasticity.
- [17] Brannon, R. M., http://www.mech.utah.edu/~brannon/public/MohrsCircle.pdf
  This is an extensive student's guide to Mohr diagrams for both symmetric and skew-symmetric tensors.
- [18] BRANNON, R.M., http://www.mech.utah.edu/~brannon/public/DerivativeRecursionTables.pdf

  This small hand-out defines the basic formulas used in gas dynamics, and provides recursion tables that you may use to convert any thermodynamic derivative into a form involving only tabulated material properties and measurable
- [19] Brannon, R.M. and Drugan, Walter J. (1993) Influence of nonclassical elastic-plastic features on shock wave existence and spectral solutions. J. Mech. Phys. Solids, 41 (2), pp. 297-330. This paper has two distinct parts. The first half is a rather arcane discussion shock admissibility conditions. The second half derives the complete set of eigenvalues and eigenvectors for the acoustic tensor associated with general plasticity, with a discussion of the implications for porous material models.
- [20] Brannon, R. M. and T. Gowen (2013) Aleatory Quantile surfaces in Damage Mechanics, *European Journal of Ceramics*, submitted November 2013.

  This paper presents an EXACT solution for stress as a function of the deformation gradient tensor for the very idealized problem of parallel fibers in a negligibly weak matrix (fibers in air). This exact solution is used to illustrate merits and flaws of common approaches to large distortion kinematics.
- [21] Brannon, R. M. (1998) Caveats concerning conjugate stress and strain measures for anisotropic elasticity. *Acta Mechanica*, Vol. 129, pp. 107-116.

  This paper presents an EXACT solution for stress as a function of the deformation gradient tensor for the very idealized problem of parallel fibers in a negligibly weak matrix (fibers in air). This exact solution is used to illustrate merits and flaws of common approaches to large distortion kinematics.
- [22] BUCK, R. CREIGHTON (1978) Advanced Calculus, McGraw-Hill.

  This is a passable reference on advanced calculus. You will need a solid foundation in advanced calculus to make good progress learning functional analysis.
- [23] BUDIANSKY, B. (1990) Tensors, in: **Handbook of Applied Mathematics 2nd Ed**, Edited by Carl E. Pearson. Van Nostrand Reinhold., pp. 179-225.
- [24] CALLEN, H.B. (1985) Thermodynamics and an Introduction to Thermostatistics, 2nd ed., Wiley.
- [25] CARTAN, E. (1966) The Theory of Spinors, Hermann Press, Paris.
- [26] CHADWICK, PETER (1976) Continuum mechanics, Concise theory and applications, 2nd Ed., Dover, Reprinted 1999.
  This book is packed with lots of nifty tidbits and unusual perspectives that will broaden your understanding of continuum mechanics and tensor analysis.
- [27] CHOU, P.C. and PAGANO, N.J. (1967) Elasticity: Tensor, Dyadic, and Engineering Approaches, Van Nostrand, Ontario Canada [also available in Dover Paperback].
- [28] DEL PIERO, G. (1979) Some properties of the set of fourth-order tensors, with application to elasticity. *J. Elasticity*, Vol. 9, No. 3. pp. 245-261.

  This paper uses a philosophy similar to ours in a discussion of basis mathematics of fourth-order linear transformations.
- [29] DESLOGE, E.A. (1982) Classical Mechanics, V. 1&2, Florida State University, Wiley & Sons, New York.
  - This wonderful two-volume book is organized as a sequence of small (usually half-page) summaries of topics in mechanics. If you need to learn a topic quickly, this book works well because you don't have to read all of it to understand these mini-chapters.



- [30] DILL, ELLIS H. (2007) Continuum Mechanics: Elasticity, Plasticity, Viscoelasticity, CRC Press, Boca Raton, FL
- [31] DIENES, J.K., (1979) On the analysis of Rotation and Stress Rate in Deforming Bodies. *Acta Mechanica* 32, pp. 217-232.
- [32] DIENES, J.K., Q.H. Zuo, and J.D. KERSHNER (2005) Impact initiation of explosives and propellants via statistical crack mechanics, *Journal of the Mechanics and Physics of Solids*.
- [33] DRUMHELLER, D.S. (1998) Introduction to Wave Propagation in Nonlinear Solids, Cambridge University Press, U.K.
- [34] DUBRULLE, A.A. (1999) An optimum iteration for the matrix polar decomposition. *Electronic Transactions on Numerical Analysis*, Vol 8, pp. 21-25.
- [35] FEDOROV, F.J. (1968) Theory of elastic waves in crystals. Plenum Press.
- [36] FLEISCH, D.A. (2012) A Student's guide to Vectors and Tensors, Cambridge University Press, UK A great (and blessedly small) book on basics of tensor analysis for non-orthonormal bases (exposing you to contravariant and covariant components, as well as curvature, Christoffel symbols, and similar notational complications that are not covered in our introductory volumes). Such concepts are important to learn early in modern physics.
- [37] FOLLAND, GERALD B. (1984) Real Analysis, Wiley-Interscience.
- [38] Friedman, Avner (1970) Foundations of Modern Analysis, Dover.
- [39] FULLER, T.J. and R.M. BRANNON (2011) On the thermodynamic requirement of elastic stiffness anisotropy in isotropic materials, *Int. J. Engr. Sci.* vol. **49**, pp. 311-321.

  This article illustrates that anisotropy induced in isotropic materials is non-negligible if the material has strongly pressure-dependent strength. The fact that isotropic materials generally have an anisotropic stiffness has long been recognized in the rubber and biomechanics communities, probably because they are able to achieve large deformations with small loads. The concept is less well understood among metals and ceramics researchers, for whom large deformations are typically achieved only in shock loading, which precludes direct measurement of stiffness. The need for induced anisotropy reveals itself indirectly when data reveal the need for a pressure-dependent shear modulus.
- [40] GEERS, M.G.D., R. DE BORST, W.A.M. BREKELMANS (1996) Computing strain fields from discrete displacement fields in 2D-solids, *International Journal of Solids and Structures*, v. 33 (29), pp. 4293-4307. http://www.sciencedirect.com/science/article/pii/0020768395002405
- [41] GELBAUM, B.R. and J.M.H. OLMSTED (1964) Counterexamples in Analysis. Holden-Day, Oakland, CA. This seminal work, mostly focused on functional analysis, drives home the power of counter-examples as a means to prove that something is NOT true.
- [42] GERALD, CURTIS F. and PATRICK O. WHEATLEY (1985) Applied Numerical Analysis, 3rd Ed., Addison-Wesley, Reading Massachusetts.
- [43] GOLDSTEIN, H., POOLE, C.P. JR; and J.L. SAFKO (2002) Classical Mechanics, Columbia University. Addison Wesley.
- [44] GOLUB, G.H., and C.F. VANLOAN (1996) Matrix Computations, 3rd ed., Johns Hopkins University Press.
- [45] GREENBERG, MICHAEL (1998) Advanced Engineering Mathematics (2nd Edition), Prentice Hall.

  A wonderful book, casually and humorously written, and packed with information that is useful to the practicing engineer. Some people prefer this edition to more recent releases. Considering how much information is in this book, it is relatively short because considerable details are "left to the student" in exercises. Consequently, get a solutions manual if you can.
- [46] Green, A.E. and W. Zerna (1968) Theoretical Elasticity, Oxford.
- [47] GURTIN, MORTON E. (1981) An Introduction to Continuum Mechanics, Academic Press, New York. This book (and a later edition with coverage of thermomechanics) has good interpretations of the principle of material frame indifference, especially on the application of momentum conservation in non-inertial frames. It's a bit choppy in its coverage of specialized topics (the assumptions that limit scope of certain chapters are not always clear).
- [48] HELNWEIN, P. (2001) Some remarks on the compressed matrix representation of symmetric second-order and fourth-order tensors. *Computer Methods in Applied Mechanics and Engineering* **190**. pp. 2753-2770.
  - This author also publishes under the name Mackenzie-Helnwein. This paper makes the important point that conventional Voigt compact matrix representations of higher-order tensors are, in fact, defining components of those tensors



- with respect to an underlying basis that is not orthonormal. Hence, Voigt components are usually either covariant or contravariant, in which the confusing factor or divisor of 2 in the Voigt system is actually a "metric."
- [49] HICKS, NOEL N. (1965) Notes on Differential Geometry, van Norstrand-Reinhold, NY 1965.
- [50] HIGHAM, N.J. (1986) Computing the Polar Decomposition with applications. *SIAM J. Sci. Stat. Comput.* 7, pp. 1160-1174.
- [51] HIGHAM, N. J. and P. PAPADIMITRIOU (1994) A parallel algorithm for computing the polar decomposition. *Parallel Computing* **20**(8), pp. 1161-1173.
- [52] HILL, R. (1950) **The Mathematical Theory of Plasticity**, Oxford University Press, New York. This is seminal work on classical plasticity.
- [53] HOGER, A. and D.E. CARLSON (1984) Determination of the stretch and rotation in the polar decomposition of the deformation gradient, *Quarterly of Applied Mathematics* **42**, no. 1, pp. 113-117. This paper points out how the Cayley-Hamilton theorem may be used to perform a polar decomposition in algebraic closed form. This paper contains errors in its formulas for finding invariants (corrected later in Ref. [80]), but the basic notions are sound.
- [54] HOLZAPFEL, G. (2000) **Nonlinear Solid Mechanics**, West Sussex, England.

  This book is very popular, especially in biomechanics, and it includes an accessible and lucid introduction to tensors and continuum mechanics (with only minor rough spots in the coverage of the principle of material frame indifference).
- [55] HORN, ROGER A., and CHARLES R. JOHNSON (1985) Matrix Analysis, Cambridge University Press, Melbourne Australia. Lucidly written, this book contains coherent (readable) proofs of theorems that most other books "leave to the reader". An excellent resource.
- [56] ITSKOV, M. (2002) The derivative with respect to a tensor: some theoretical aspects and applications. ZAMM-Z Angew. Math. Mech. 82(8), pp. 535-544.

  This paper has some useful discussions, but it fails to use self-defining notation. For example, it uses the cross product symbol to denote dyadic multiplication, while it uses the dyadic multiplication symbol to denote what we call "leafing" multiplication. It claims as "new" some operations and results that had been around for twenty or more years earlier.
- [57] ITSKOV, M. (2000) On the theory of fourth-order tensors and their applications in computational mechanics. *Comput. Methods Appl. Mech. Engrg* **189**, pp. 419-438.
- [58] JoG, C. S. (2006) A Concise Proof of the Representation Theorem for Fourth-Order Isotropic Tensors. *Journal of Elasticity* **85**(2), pp. 119-124.
- [59] KREYSZIG, E. (1989) Introductory Functional Analysis with Applications, John Wiley & Sons.
- [60] LAI, W. M., D. RUBIN, and E. KREMPL(2010) Introduction to Continuum Mechanics, 4th Ed. Butterworth-Heinemann.
  New students to the subject seem to like this book, which is surprising since it uses somewhat old-fashioned (and certainly not self-defining) notation. For a continuum book, perhaps excessive coverage is given to small-deformation analysis methods that are taught in undergraduate materials mechanics courses.
- [61] Leighton, J.T., C.R. Champion, and L.B. Freund (1987) Asymptotic analysis of steady dynamic crack growth in an elastic-plastic material, Journal of the Mechanics and Physics of Solids 35, pp. 541-63.
- [62] Lin, R. (2003) Hypoelasticity-based analytical stress solutions in the simple shearing process, *Journal of Applied Mathematics and Mechanics* **83**, pp. 163-171
- [63] Lu, J. and P. Papadopoulos (1997) On the Direct Determination of the Rotation Tensor from the Deformation Gradient, *Math. Mech. Sol.*, **2**, pp. 17-26.
- [64] LEBEDEV, L.P. and M.J. CLOUD (2003) **Tensor Analysis**, World Scientific, NJ.

  This book is a pretty good introduction to tensor analysis. This book would be comprehensible to undergraduates, which alone makes it worthy since so many of the books out there can only be read by mathletes. This book's has a few quirks, but nothing outrageously bad. The book includes a very nice introduction to differential geometry. It covers curvilinear coordinates too.
- [65] MALVERN, LAWRENCE E. (1969) Introduction to the Mechanics of a Continuous Medium, Prentice-
  - This is one of the best known references on elementary continuum mechanics. It is difficult for most people to learn continuum mechanics for the first time from this book, but after reading a simpler book first (e.g., Mase-Smelser-Mase), Malvern's book can be an indispensable reference.



- [66] MASE, G.T., R.E. SMELSER, and G.E. MASE (2009) Continuum Mechanics for Engineers 3rd Ed, CRC Press, Boca Raton, FL.
  - This very readable introduction serves as an excellent starting point for newcomers to continuum mechanics. Like many continuum textbooks, it is somewhat dry with generic sketches (to convey generality of the concepts), but it would benefit from a few more specific applications.
- [67] MEHRABADI, M.M. and S.C. COWIN (1990) Eigentensors of linear anisotropic elastic materials. *Quarterly Journal of Mechanics and Applied Mathematics* 43, pp. 15–41.
- [68] MORAWIEC, A. (2004) Orientations and Rotations: Computations in Crystallographic Textures, Springer-Verlag, Berlin.
  This book might be a bit difficult for an engineer to read, but after you use Ref. [15] to learn the basics, you will find
- [69] MUSKHELISHVILI, N. I. (2008) Singular Integral Equations: Boundary Problems of Function Theory and Their Application to Mathematical Physics, Dover (reprint of work originally published 1946).
- [70] NEMAT-NASSER, S. (2004) Plasticity: A Treatise on Finite Deformation of Heterogeneous Inelastic Materials, Cambridge Monographs on Mechanics.
- [71] NGUYEN, H., J. BURKARDT, M. GUNZBURGER, and L. Ju (2009) Adaptive anisotropic meshes for steady-state convection dominated equations, *Comp. Meth. Appl. Mech. Engrg.* 198, pp. 2964-2981.
- [72] NYE, J.F. (1957) Physical Properties of Crystals, Oxford Clarendon Press.

proper rigorous discourse about the same topics in this book.

- [73] NOBLE, BEN, and JAMES W. DANIEL (1988) **Applied Linear Algebra, 3rd Ed.**, Prentice-Hall. This is a classic linear algebra textbook (especially used in the 1980s and 1990s).
- [74] PARKER, SYBIL [Ed.] (2002) McGraw-Hill Dictionary of Scientific and Technical Terms 6th Edition, McGraw-Hill Professional. The definitive resource for exactly what its title says, though the pronunciation guidance in this edition is a bit prescriptive with pronunciations (stating how things should be pronounced instead of how they actually are pronounced by most practitioners).
- [75] PEARSON, CARL (editor) (1990) Handbook of Applied Mathematics, 2nd Ed., Van Nostrand Reinhold.
  - This book uses the same "no symbol" notation for dyadic multiplication that we espouse. This book also has excellent succinct chapters on matrix and vector analysis.
- [76] RAMKRISHNA, D. and N. R. AMUNDSON (1985) Linear Operator Methods in Chemical Engineering, Prentice-Hall.
  - Don't let the word "chemical" dissuade you from looking at this book. Here, you will find proper statements of key theorems from analysis that apply to all branches of engineering.
- [77] RASHID, M (1993) Incremental kinematics for finite-element applications. *Int. J. Numerical Meth. in Engr.* **36**, no.23, pp. 3937-3956.
- [78] RUDIN, WALTER (1987) Real and Complex Analysis, McGraw-Hill.
  For engineers, this book is extraordinarily difficult to read. However, if you ever want a truly proper statement of a theorem, along with a careful list of underlying assumptions and mathematical domains of applicability, then this is the place to look.
- [79] RYCHLEWSKI, J. (1984) On the evaluation of anisotropy of properties described by symmetric second-order tensors, *Czech J. Phys*, **Vol. 34**, pp. 499-506.
- [80] SAWYER, KEN (1986) Comment on the paper 'Determination of the stretch and rotation in the polar decomposition of the deformation gradient' by A. Hoger and D.E. Carlson, *Quart. Appl. Math* 4, pp. 309-311.
  - This paper corrects errors in Ref. [53].
- [81] SCHWER, L.E. (2006). Guide for verification and validation in computational solid mechanics, American Society of Mechanical Engineers, ASME V&V 10-2006.

  This ASME standard essentially formalizes the distinction between good mathematics, in which you are solving the equations correctly, and good physics, in which you are solving the correct equations.
- [82] SIMMONDS, JAMES G. (1994) A Brief on Tensor Analysis, Springer-Verlag, NY.
- [83] SIMO, J.C. and T.J.R. HUGHES (1998) Computational Inelasticity (Interdisciplinary Applied Mathematics, Vol. 7), Springer-Verlag.
  - Both Simo and Hughes are brilliant mathematicians. They approach every problem with impeccable ratiocination. You



will find reliable algorithms in this book. Unfortunately, however, this work is chronically lacking in basic everyday insight. For example, they provide an algorithm for the polar decomposition that requires computation of some initial "helper" variables. NOT ONCE do they inform the reader that these helper variables are just the standard  $J_2$  and  $J_3$  invariants. Their derivations are needlessly complicated. In their efforts to always present at least second-order accurate integration algorithms, they neglect to discuss the subtle issues of even first-order algorithms.

- [84] SIMO, J.C. and PISTER, K.S. (1984) Constitutive Relations for Thermo-elastic Porous Solids within the Framework of Finite Deformations, Comp. Meth. in Appl. Mech. and Engr., 46, pp. 201-215.
- [85] SLOAN, S.W. and J.R. BOOKER (1986) Removal of Singularities in Tresca and Mohr-Coulomb Yield Functions, *Communications in Applied Numerical Methods*. Vol. 2, pp. 173-179.
- [86] SMART, W.M. (1977) Textbook on Spherical Astronomy. 6th Ed. revised by R. M. Green., Cambridge University Press.
  Contains theorems that are useful in generating the mesh of the sphere.
- [87] SMITH, G.F. (1971) On isotropic functions of symmetric tensors, skew-symmetric tensors, and vectors, *Int. J. Eng. Sci.*, Vol. 9, pp. 899-916.
- [88] SMITH, G.F., M.M. SMITH, and R.S. RIVLIN (1963) Integrity bases for a symmetric tensor and a vector The crystal classes. *Arch. Rational Mech. Anal.*, Vol 12.

  The title says it all. This paper shows the most general quadratic form that can be constructed from the isotropic invariants for a crystal class.
- [89] SPENCER, A.J.M. (1971) Theory of Invariants, in Continuum Physics I, edited by A.C. Eringen Academic, New York.
  Derives the integrity bases for arbitrary numbers of vectors and tensors for (i) full orthogonal group (ii) proper orthogonal (rotation) group, (iii) transverse isotropy, and (iv) crystal classes.
- [90] STAKGOLD, IVAR (1979) **Green's Functions and Boundary Value Problems**, Wiley.

  This is a difficult book for the typical engineer to read because Stakgold rarely spells out the *meaning* of the equations. If, however, you are adept enough to figure this out yourself, then the effort to plow through this book is worth it because, by the end, you will think about functions and transformations in an entirely new way. You will be left with the habit of seeing *everything* as an operation (which, by the way, is the premise underlying Mathematica), and you will routinely apply Stakgold's operation categorization
- [91] STEWART, G.W. (1973) **Introduction to Matrix Computations**, in *Computer Science and Applied Mathematics series of monographs and textbooks*. Academic Press.

  Great resource for computational linear algebra!
- [92] STRANG, G. (1986) Introduction to Applied Mathematics, Wellesley-Cambridge Press, Wellesley, MA.
  Because of its many physical examples, this is a good engineer's and physicist's reference.
- [93] TAI, CHEN-TO (1997) Generalized Vector and Dyadic Analysis. Applied Mathematics in Field Theory, 2nd Ed. IEEE Press, NY.
- [94] TARANTOLA, A. (2006) Elements for Physics: Quantities, Qualities, and Intrinsic Theories. Berlin, Springer-Verlag.
- [95] TCHONKOVA, M. and S. STURE (2001) Classical and Recent Formulations for Linear Elasticity, *Archives of Computational Methods in Engineering*, Vol. 8, 1, 41-74.

  This is a very nice summary paper.
- [96] THOMSON, PHILIP A. (1972) Compressible Fluid Dynamics, McGraw-Hill, USA.

  To our knowledge, this is the earliest reference that points out that Reynolds transport theorem is nothing more than a specific instance of the (more general) Leibniz theorem. Even modern textbooks fail to mention this illuminating fact.
- [97] TING, TOM (1985) Determination of  $C^{1/2}$  and  $C^{-1/2}$  and more general isotropic tensor functions of C, J. *Elasticity* **15**, pp. 319-323.

  This paper is relevant to computation of the polar decomposition, which requires the "square root" of a tensor.
- [98] TRUESDELL, C., W. NOLL, and S.S. ANTMAN. (2004) The nonlinear field theories of mechanics, Vol. 3, Springer.
- [99] VARADAN, VASUNDARA V., JIANN-HWA JENG, and VIJAY K. VARADAN (1987) Form invariant constitutive relations for transversely isotropic piezoelectric materials. *J Acoust. Soc. Am.* **82** (1) pp. 337-342. This paper shows how to derive transversely isotropic constitutive relations using integrity basis for the transverse orthogonal group of transformations.



- [100] VIGGIANI, G., C. TAMAGNINI, F. CALVETTI. (2005) An assessment of plasticity theories for modeling the incrementally nonlinear behavior of granular soils. *J. Engineering Mathematics*, v.52, pp. 265-291.
- [101]WALPOLE, L.J. (1984) Fourth-rank tensors of the thirty-two crystal classes: Multiplication tables, *Proc. R. Soc. Lond. A* 391, pp. 149-179.
- [102] WOLFRAM, STEPHEN (1999) **The Mathematica Book, 4th Ed**, Cambridge Univ. Press.

  Even though *Mathematica* caters to mathematicians, the basic premise upon which this symbolic math program is based (namely, that *everything* is an operation) is so useful that engineers would be well served to embrace it as well
- [103] WYLIE, C. RAY, and LOUIS C. BARRETT (1982) **Advanced Engineering Mathematics**, McGraw-Hill. For engineers, this is a great resource book. It has summaries of numerous branches of mathematics, along with brief explanations of how to apply the mathematics to engineering problems. Consider using this book in conjunction with Refs. 3, 45, and 75.
- [104] YOUSSEF M. A. HASHASH, J. I. C. Y. D. C. W. (2003). Glyph and hyperstreamline representation of stress and strain tensors and material constitutive response. *International Journal for Numerical and Analytical Methods in Geomechanics* 27(7): 603-626.

## [105] The Way of Analysis

- [106]MARIS, A.W. (1967) Two definitions of gradient, J. Appl. Mechanics, 34E: 504-506.
- [107]ZHENG, Q-S (1994) Theory of representations for tensor functions a unified invariant approach to constitutive equations, *Appl. Math. Rev.*, 47(11):545-587